

# Stochastic

25/10/2015

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Distributive law: -  $\rightarrow$  Sheet 2  $\leftarrow$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Demorgan's Law

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

Theorem 3:-

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Ex: prove that  $P(A^c) = 1 - P(A)$

$$A^c \cup A = S$$

$$P(A^c \cup A) = P(S)$$

$$P(A) + P(A^c) - P(A \cap A) = 1$$

$$P(A^c) = 1 - P(A)$$

Q1: a) A but not B  $\Rightarrow A - B, A - (A \cap B)$

b) Either A or B but not both

$$\Rightarrow (A \cup B) - (A \cap B)$$

$$, (A - B) \cup (B - A)$$

b) A or not b

$$\Rightarrow A \cup B^c, (B - A)^c$$

Q2: a) A and B but not C

$$\Rightarrow (A \cap B) - C$$

b) only A

$$\Rightarrow A - (B \cup C)$$

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Q3: Coin and Die

a) A: head and even number

B: prime number

C: tails and odd no.

$$S = \{(H, 1), (H, 2) \dots (T, 1), (T, 2) \dots\}$$

$$A = \{(H, 2), (H, 4), (H, 6)\}$$

$$B = \{(H, 2), (H, 3), (H, 5), (T, 2), (T, 3), (T, 5)\}$$

$$C = \{(T, 1), (T, 3), (T, 5)\}$$

b) A or B

$$\{(H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 2), (T, 3), (T, 5)\}$$

c) only B

$$B - (A \cup C) = (B - A) \cup (B - C)$$

$$= \{(H, 3), (H, 5), (T, 2)\}$$

$$d) P(H) = 2 P(T)$$

$$P(H) + P(T) = 1$$

$$2 P(T) + P(T) = 1 \Rightarrow P(T) = 1/3$$

$$P(H) = 2/3$$

Q7:- 10 boys, 20 girls

half boys and half girls have brown eyes

$$P(\text{a boy or has brown eyes}) = P(E_1 \cup E_2)$$

$E_1$ : a boy

$$P(E_1) = 1/3$$

$E_2$ : brown eyes

$$P(E_2) = 5/30 = 1/6$$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= 1/3 + 1/6 - 1/6$$

$$= 2/3$$

Theorem

$$S = \{ (c_1, c_2, c_3, \dots), (c_1, c_2, c_3, \dots) \}$$

$$P(\text{sample point}) = P(c_1) P(c_2) P(c_3) \dots$$

Q10: The odds that an event will occur are a:b

$$\frac{a}{a+b}$$

Q11: 3 to 2

$$P(\text{Event}) = \frac{3}{5} ;$$

Q 18: Die, Probability of a number appearing is proportional to the number

$$P(1) = 1/21, P(2) = 2/21$$

$$P(3) = 3/21$$

A: even number

$$P(A) = \frac{12}{21} = \frac{2}{21} + \frac{4}{21} + \frac{6}{21}$$

Q 19: Prove that  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$$\begin{aligned} P(A \cup (B \cup C)) &= P(A) + P(B \cup C) - P(A \cap (B \cup C)) \\ &= P(A) + P(B) + P(C) - P(B \cap C) \\ &\quad - P((A \cap B) \cup (A \cap C)) \end{aligned}$$

$$\downarrow$$
$$- \left[ \begin{array}{l} P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \end{array} \right]$$

5, 6, 8, 12, 13, 15, 20  $\Rightarrow$  Report



\* Conditional probability:-

the conditional probability of an event A given that B has occurred is denoted by:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Sheet 3:-

- ④ 25% failed in math  
15% failed in chemistry  
10% failed in both

a) if he failed in chemistry, what is the probability that he failed in math.

$$E_1: \text{Math}, P(E_1) = 0.25$$

$$E_2: \text{Chem.}, P(E_2) = 0.15$$

$$P(E_1 \cap E_2) = 0.1$$

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{0.1}{0.15} = \frac{10}{15} = \frac{2}{3}$$

b) if he failed in math, what is the probability that he failed in chemistry

$$P(E_2|E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{2}{5}$$

⇒ Turn over

c) fail in math or chem.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= 0.25 + 0.15 - 0.1 = 0.3$$

Q5: Let A and B

$$P(A) = 1/2, P(B) = 1/3, P(A \cap B) = 1/4$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 3/4$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 7/12$$

$$P(A^c|B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} \Rightarrow P(A \cup B)^c = 1 - \frac{7}{12} = 5/12$$

$$= \frac{5/12}{2/3} = 5/8$$

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Q6:  $P(A) = 3/8, P(B) = 5/8$

$$P(A \cup B) = 3/4$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{5/8} = 2/3$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{3}{8} + \frac{5}{8} - \frac{3}{4} = \frac{1}{4}$$

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\* Independent events:-

$$P(B) = P(B|A)$$

Ex: if  $A, B$  are independent

$$- P(A \cap B) = P(A) P(B)$$

$$- P(B|A) = P(B)$$

$$- \frac{P(A \cap B)}{P(A)} = P(B)$$

Q1: if  $A, B$  are independent events, prove that  $A^c$  and  $B^c$  are independent.

$$P(A \cap B) = P(A) P(B)$$

$$P(A^c \cap B^c) = P(A \cup B)^c$$

$$= 1 - P(A \cup B)$$

$$= 1 - P(A) - P(B) + P(A \cap B)$$

$$= \underbrace{1 - P(A)} - P(B) + P(A) P(B)$$

$$= 1 - P(A) - P(B)(1 - P(A))$$

$$= (1 - P(A))(1 - P(B))$$

$$= P(A^c) P(B^c)$$

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Same problem for  $A, B^c$  (Prove)

$$P(A \cap B^c) = P(A)P(B^c) \leftarrow \text{المطلوب}$$

$$\begin{aligned} P(A \cap B^c) &= P(A) - P(A \cap B) \\ &= P(A) - P(A)P(B) \\ &= P(A)(1 - P(B)) \\ &= P(A)P(B^c) \end{aligned}$$

Q2: 3 fair coins, Find the probability that they are all heads if:

a) the first coin is head

b) one of the coins is head

$$S = \{ (h, h, h), (h, h, t), \dots \}$$

$E_1$ : all heads

$$P(E_1) = 1/8$$

$E_2$ : first head

$$P(E_2) = 1/2$$

$$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1/8}{1/2} = 1/4$$

$E_3$ : one is head

$$P(E_3) = 7/8, \quad P(E_1 \cap E_2) = 1/8$$

$$P(E_1 | E_3) = \frac{P(E_1 \cap E_2)}{P(E_3)} = \frac{1/8}{7/8} = 1/7$$